VERSOR INVESTMENTS

Has Trend Gone Flat? Return Convexity in Trend Following

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Firm Overview

Versor Investments is a quantitative investment boutique where data, innovation, and market expertise drive every decision. Headquartered in New York, Versor's investment team merges decades of quantitative research with an ethos that fosters ingenuity and innovation.

Leveraging modern statistical methods and vast datasets, Versor Investments works to create diversified sources of absolute returns across multiple asset classes. Alpha forecast models, portfolio construction, and the trading process rely on the ingenuity and mathematical expertise of 40+ investment professionals, which is underpinned by a rigorous scientific, hypothesis-driven framework. Versor implements state of the art technology infrastructure for risk management, portfolio optimization, and trade execution, developed over 200+ human work years.

Versor upholds client interests with 100% employee ownership and substantial co-investment from partners. Versor offers two categories of investment products: Hedge Funds and Alternative Risk Premia. Both product lines are designed to provide superior risk-adjusted returns while exhibiting low correlation to traditional and alternative asset classes.

Has Trend Gone Flat? Return Convexity in Trend Following

Executive Summary

Historically, the returns generated by CTA or "trend-following" hedge funds have exhibited an unusual and attractive combination of high average returns and positive convexity.

We document that, over time, the returns of CTA funds have lost some of both of these appealing characteristics, especially return convexity. The long-term trend signal favored by the largest CTA funds have lost more convexity than short-term trend signals that require faster trading.

Any portfolio that successfully implements time-series forecasts exhibits positive convexity and positive returns: A successful time-series signal tends to predict positive returns when markets rise and leads to profitable long positions. By the same token, such a signal tends to predict negative returns when markets fall and leads to profitable short positions.

Based on this logic, we show how to enhance the performance characteristics of trend-following portfolios with additional, newer signals not based on past trends. We demonstrate that such portfolios have been able to improve upon the positive return convexity previously associated with CTA strategies.

Interestingly, cross-sectional signals without market exposure can also provide return convexity. A primary source of convexity for cross-sectional signals is higher returns during volatile periods with large market returns. Strategies that generate larger returns during volatile periods than during calm periods are likely to have positive convexity.

Investors looking for positive convexity and positive returns during less volatile times were previously attracted to trend-following. They should now consider enhanced portfolios that also implement non-trend signals to enhance portfolio convexity.

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1 Introduction

Historically, the returns generated by CTA or "trend-following" hedge funds have exhibited an unusual and attractive combination of high average returns and positive convexity. As a result, CTA funds have been able to generate positive returns in periods with large positive and large negative equity market returns without incurring offsetting negative returns during more normal market environments. As Fung and Hsieh (2001) document, this payoff profile resembles being long options, which earn positive returns during volatile periods, or purchasing a form of portfolio insurance – but without the premium costs associated with insurance strategies.

Many investors find it challenging to maintain allocations to insurance strategies if those strategies earn negative returns during extended calm periods. Of course, that also means that these investors go without the benefit of the insurance during periods of market stress. Historically, trend-following allocations have been easier to maintain because they did not charge an obvious premium.

We document that – over time – the returns of CTA funds have lost some of both of these appealing characteristics. Clearly, that makes CTA returns less appealing than they used to be. We can link the returns of CTA funds to trend-following signals and show that the average returns and convexity associated with these signals has declined over time. In particular, the long-term trend signals favored by the largest CTA funds that make up the SG Trend index have lost more convexity than shorter-term trend signals that require faster trading.

While it is possible that these declines in convexity and average returns are temporary, they have now lasted for many years. This must give rise to concerns that a secular change has affected the returns to trend-following strategies.

Basic logic implies that any successful time-series portfolio should exhibit positive convexity and positive returns. A successful time-series signal tends to predict positive returns when markets rise and leads to profitable long positions. By the same token, such a signal tends to predict negative returns when markets fall and leads to profitable short positions. Note that a successful time-series does not have to be always correct about the sign of the return. If a time-series signal has predictive power on average, it should lead to positive returns with positive convexity.

Interestingly, cross-sectional signals without market exposure can also provide return convexity. A primary source of convexity for cross-sectional signals is better performance during volatile periods with large market returns. Strategies that generate larger returns during volatile periods than during calm periods are likely to have positive convexity. Although such "long gamma" strategies are relatively easy to construct with options, these option strategies generally involve upfront premium payments that reduce their profitability. During extended calm periods, these premium costs can lead to material losses, which in turn can lead investors to abandon the strategy. Due to their premium costs and the associated challenges of maintaining allocations, we do not consider options-based strategies.

Based on this logic, we stress the importance of enhancing the performance characteristics of trend-following portfolios with additional, newer signals that enhance return convexity. We show how to measure return convexity and how to construct portfolios with attractive convexity.

The remainder of the paper proceeds as follows: Section 2 demonstrates that CTA fund returns are largely driven by trend-following signals; section 3 describes how to measure the convexity of portfolio returns; section 4 shows that, over time, the returns to trend-following signals have become smaller and less convex; section 5 introduces additional signals that have had higher and more convex returns than trend following signals; section 6 introduces a framework for trading off average performance against convexity; section 7 illustrates portfolio combinations of signals with strong returns and high convexity; section 8 describes performance of trend-following strategies in 2021; and section 9 concludes.

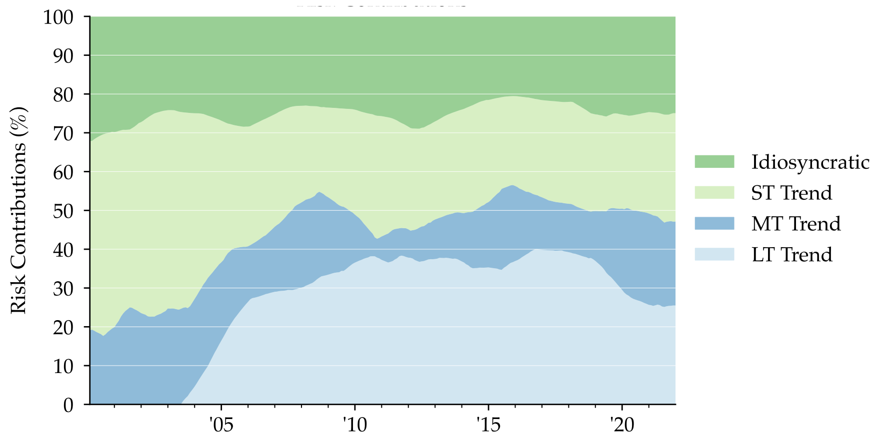
2 CTA Hedge Funds and Trend-Following

While it is generally accepted that CTA hedge fund returns are driven by trend-following strategies, many investors underestimate how central these strategies are to CTA funds. We show that more than 90 percent of return variation for CTA funds can be explained by a blend of short-, medium-, and long-term trend-following signals in futures markets covering commodities, equity indexes, fixed income, and currencies.

Figure 1 shows the results of returns-based style analysis for the SG Trend index, which is an average of the returns for the 10 largest CTA hedge funds.[1] The style analysis blends short-, medium-, and long-term trend-following strategies to find the portfolio returns that most closely resemble the return to the SG Trend index. The estimation allows the exposures to the different investment strategies to vary over time, in case managers join or leave the index or in case continuing managers change their investment style.

[1]See Sharpe (1992) for a description of returns-based style analysis.

Figure 1: CTA Investment Styles



The figure shows risk contributions to the SG Trend index from different investment styles. The SG Trend index is composed of the 10 largest CTA funds. The estimates stem from return-based style analysis of monthly SG Trend index returns from January 2000 to December 2021. The investment styles include simulated long-term, medium-term, and short-term trend-following. The simulated trend-following strategies invest in roughly 100 futures contracts across the 4 major asset classes: equities, fixed income, commodities, and currencies.

The idiosyncratic contributions contain risk from SG Trend index return components that we cannot attribute to trend-following.

The replicating portfolios consisting of the simulated trend-following returns capture about 70 to 80 percent of the return risk of the SG Trend index returns. That corresponds to a return correlation of approximately 90 percent.
Source: Data received from Société Générale. Internally prepared by Versor Investments.

As the figure shows, trend-following investment strategies account for approximately 75 percent of the overall return risk in the SG Trend index, leaving relatively little room for other investment styles among the CTA funds included in the index. Equivalently, the returns of the pure trend-following replicating portfolio have a correlation of nearly 0.9 with the SG Trend index. Among the trend-following styles, long-term trend-following now contributes the largest amount of risk. This contrasts with the large risk share of short-term trend-following styles in the early 2000s, when many of the included CTA funds were much smaller. We attribute this preference for long-term trends among the largest CTA funds to the potentially large market impact associated with the faster trading required for short-term trend signals. Very large CTA funds likely find it prohibitively expensive to implement these short-term trends.

2.1 Simulated Trend-Following

The trend-following strategies we consider use time-series momentum and moving-average cross-over signals. The signals are based on lookback periods from 1 to 12 months. The long-term trend signals use lookback periods between 6 and 12 months. The medium-term trend signals use lookback periods between 3 and 6 months. The short-term trend signals use lookback periods less than 3 months.

We simulate the strategies and their returns by investing in liquid futures contracts across the 4 major asset classes: equity index futures, fixed-income futures, commodity futures, currency futures and forwards. There are about 100 futures contracts in total, fairly evenly split across the asset classes.

We construct trend signals based on past returns. Within each asset class, we allocate similar risk to all contracts by deflating positions by the risk of the corresponding contract. Finally, we use equal long-term risk budgets to allocate across asset classes.

3 Measuring Convexity in Returns

Convexity measures a nonlinear response in investment returns with respect to a reference returns. We focus on equity market returns as the reference returns. This focus is natural for investors with large risk allocations to equity markets.

3.1 Convexity with Respect to Equity Market Returns

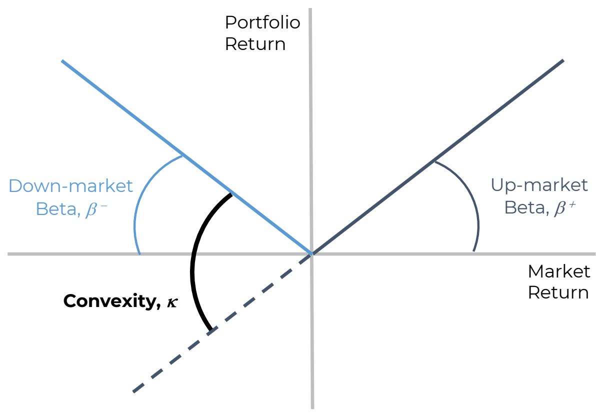
The main measure of market exposure is beta, measuring the linear dependence of portfolio returns on market returns,

$$r_t = \alpha_0 + \beta_0 r_{m,t} + \epsilon_t. \tag{1}$$

Throughout the analytical discussion, we use returns measured in excess of the risk-free rate, $r_{f,t}$. Here, r_t is a portfolio excess return, $r_{m,t}$ is the excess return on the market portfolio, β_0 measures the portfolio's market exposure, α_0 is the portfolio's average excess return not attributable to market exposures, and ϵ_t is an unidentified return contribution in period t. We generally estimate the coefficients α_0 and β_0 via regression.

Clearly, the market exposure captured by β_0 is constant over time. Also, it does not vary with market returns. Jensen (1972) and Henriksson and Merton (1981) discuss that such constant measures of market exposure cannot identify whether a portfolio manager or investment strategy has market timing ability. To allow for market timing, we can model different

Figure 2: Convexity in Returns



The figure illustrates the effects of market timing with higher exposures, β^+, during periods with positive market returns and lower exposures, β^-, during periods with negative market returns. We define convexity as the difference between the up-market and down-market exposures, $\kappa = \beta^+ - \beta^-$.
Source: Internally prepared by Versor Investments.

market exposures, depending on whether market returns are positive or negative,

$$r_t = \alpha + \beta^+ r^+_{m,t} + \beta^- r^-_{m,t} + \epsilon_t. \tag{2}$$

As described by Henriksson and Merton (1981), β^+ measures the average market exposure during periods with positive market returns $r^+_{m,t} \geq 0$ and β^- measures the average market exposure during periods with negative market returns $r^-_{m,t} \leq 0$. As before, we can estimate these coefficients using regressions. In these regressions, the independent variables are $r^+_{m,t} \equiv \max\{0, r_{m,t}\}$ and $r^-_{m,t} \equiv \min\{0, r_{m,t}\}$.[2]

As in a standard market regression, the intercept term α indicates the average portfolio return conditional on zero excess returns for the equity market. The regressions impose the constraint that this excess return is the same during positive and negative market regimes. This restriction attributes return variation across market regimes to differences in market exposures, not differences in idiosyncratic returns. Freely estimating separate intercepts

[2]A constraint built into this direct approach is that we pre-determine the breakpoint between positive and negative equity environments. The point of zero excess return is a natural division between positive and negative equity regimes. However, the approach generalizes to any process that identifies positive and negative market regimes, so that that we can use the appropriate returns in the regression. For example, one could use the Hamilton (1989) regime-switching framework to determine market regimes. If the two market regimes have different volatilities, it may be appropriate to use generalized least squares estimation methods.

and exposures in both market regimes is unattractive because the different intercepts produce potentially large discontinuities in expected portfolio returns at $r_{m,t} = 0$.

The difference in market exposures during up markets and down markets is a natural measure of return convexity:

$$\kappa \equiv \beta^+ - \beta^-. \tag{3}$$

Figure 2 illustrates this definition of convexity in returns. A portfolio that displays constructive market timing has less market exposure during down markets than during up markets: $\beta^- < \beta^+$. Ideally, we might wish for positive exposures during up markets, $\beta^+ > 0$, and negative exposures during down markets, $\beta^- < 0$. However, as long as the difference in market exposures is positive, the portfolio exhibits positive convexity and constructive market timing.

Importantly, this measure separates convexity from average or overall market exposures. There may be cases where we wish to manage the average beta to a particular value, like 0 or 1. For any choice of overall market exposure, β, it seems clear that investors would prefer a portfolio with more convexity, "all else equal". A portfolio with more convexity provides better insurance during periods of poor market returns.

Note that keeping the other portfolio characteristics equal as we vary convexity, may correspond to changes in the regression coefficients. For example, the expected excess return on the portfolio is

$$\mu = \alpha + \beta^+ \mu_m - \kappa \mu_m^-,$$

where μ_m is the average excess return on the equity market and $\mu_m^- = E r_m^-$ is the average value of r_m^-. Since $\mu_m^- \leq 0$, increasing convexity directly increases the average return on the portfolio. To maintain a constant average return as convexity rises, expected portfolio returns at $r_{m,t} = 0$, given by α, have to decline. Of course, if an investment strategy can increase convexity and average returns, all the better.

By defining convexity as the difference between two betas we ensure that convexity inherits some useful properties familiar from market betas. First, the convexity of a portfolio is equal to the corresponding weighted average of convexities of the constituent assets. Second, because convexity is a difference of betas, the market portfolio has zero convexity with respect to its own returns. Third, the risk free asset has zero convexity. Fourth, convexity is proportional to leverage. These properties imply that we can use hedge positions in the market portfolio in order to remove market beta from the portfolio without affecting the portfolio's convexity.

As for linear market exposures, there are applications where we prefer a measure that does not depend on leverage. In these situations, correlation provides a scale-free measure of linear exposure. Since convexity is the difference of two linear exposures, the scale-free measure of convexity is

$$\nu = \kappa \, \frac{\sigma_m}{\sigma}, \tag{4}$$

where κ is the convexity measure from equation 3, σ_m is the standard deviation of market returns and σ is the standard deviation of the portfolio returns.

An equivalent representation of the market timing regression in equation 2 that can be analytically more convenient is

$$r_t = \alpha + \beta^+ r_{m,t} - \kappa r_{m,t}^- + \epsilon_t. \tag{5}$$

Estimating this form via standard regression methods conveniently produces standard errors for the convexity estimate.

If we estimate a standard market regression, like equation 1, for a portfolio with a convex investment strategy, the regression produces a beta estimate

$$\hat{\beta} \approx \beta^+ - \frac{\kappa}{2}, \tag{6}$$

if the distribution of market excess returns $r_{m,t}$ is approximately symmetric about 0.[3]

3.2 Convexity and Return Horizon

Like market beta, convexity estimates can vary with return horizons. This is a property of return covariances. Moreover, if convexity is generated by dynamic trading strategies, like trend following, then slower strategies may display little convexity over short return horizons but material convexity over long return horizons. Obviously, "long" and "short" return horizons must be considered relative to the speed of the trading strategy.

Given the typical speeds of trend following signals and the other signals we investigate, we focus on monthly returns in our empirical analysis. We

[3] This result follows directly from the standard omitted variable bias calculations for linear regressions. (See Wooldridge (2010), for example.) The bias term is $-\kappa\,\mathrm{Cov}(r_{m,t}, r_{m,t}^-)\,/\,\mathrm{Var}(r_{m,t})$. Inspection reveals that the covariance calculation produces zeros for all $r_{m,t} \geq 0$ and standard variance terms for all $r_{m,t} \leq 0$. By symmetry, the covariance is half of the variance of the market excess returns.

have also analyzed quarterly and annual returns. For the signals we use here, quarterly and annual return horizons produce qualitatively similar results.

Longer return horizons reduce the number of available non-overlapping observations. Such a reduction in sample size generally reduces statistical significance of results. This can be offset by using overlapping return periods while making the appropriate adjustments to the statistical estimates. To avoid these complications, we focus on non-overlapping monthly returns in our empirical analysis.

3.3 Convexity with Respect to Other Returns

There may be scenarios where we would like to measure convexity with respect to other returns. Mechanically, this is straightforward. We simply replace the market excess return on the right side of the regressions, $r_{m,t}$, with the excess return of interest. Everything else stays the same.

For an investment strategy that tries to time a particular asset, a natural reference return is the long-only excess return to that asset. Any successful timing strategy should have positive convexity with respect to the return of the traded asset: the strategy should be less long during periods of negative asset returns. In this context, testing for positive convexity is equivalent to testing for timing skill.

For an investment strategy that operates in a particular asset class, an natural reference return is the return to that asset class. For example, an investment strategy that attempts to time the bond market should have positive convexity with respect to a bond market benchmark return.

3.4 Convexity and Skewness

If the reference returns $r_{m,t}$ and forecast errors ϵ_t are symmetrically distributed, then an investment strategy with positive convexity generates returns with positive skewness.[4] Barberis and Huang (2008) and Harvey and Siddique (2000) argue that investors find positively skewed returns appealing.

Unfortunately, it becomes harder to link convexity to skewness if the reference returns are not symmetric. Bessembinder (2018) and Albuquerque (2012) show that returns for individual stocks generally have strong positive skewness; Campbell and Hentschel (1992) and Albuquerque (2012) show that daily and monthly returns for equity market indexes have negative skewness. However, Kim and White (2004) argue that standard estimates

[4] Potters and Bouchaud (2005) and Martin and Zou (2012) derive the positive skewness induced by trend-following trading strategies for unpredictable asset returns.

of higher moments are easily contaminated by outliers and that robust estimates of skewness for equity market index returns show less evidence of asymmetry in the return distribution.

Although negative skewness of equity market returns would make it difficult to say how much convexity is required for a market-timing strategy to have positively skewed returns, positive convexity generally implies less negatively skewed returns.

4 Trend-Following Over Time

Using both the SG Trend returns and the simulated returns to the trend-following signals, we now show that the convexity and average returns to these strategies have declined over time.

We estimate convexity κ for the different trend-following strategies using regressions. By running these regressions for rolling estimation periods, we can investigate how convexity has changed over time.

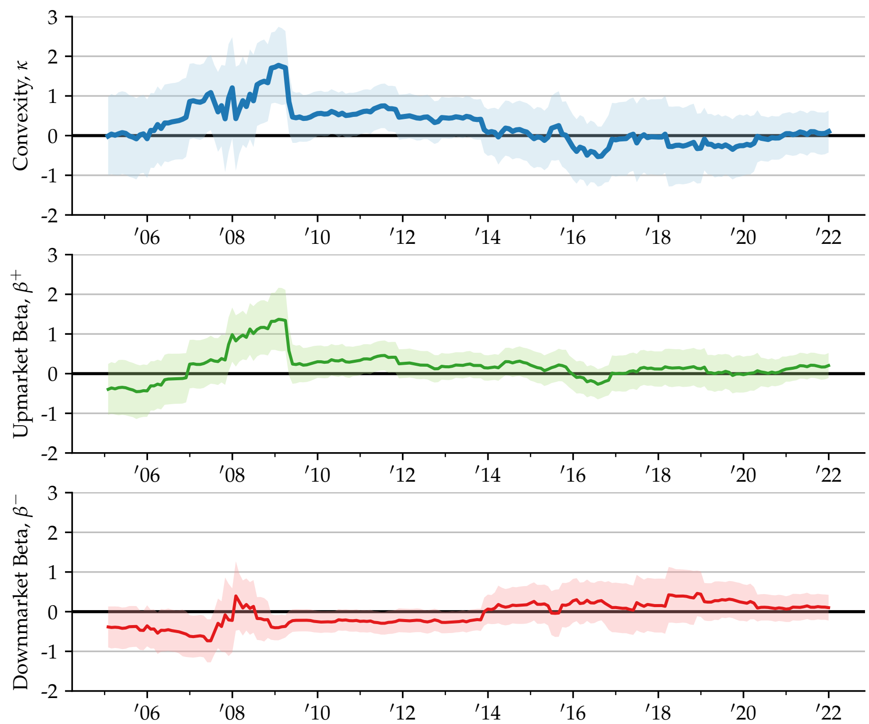
We measure equity market returns using the S&P 500 index. The results are qualitatively similar for other broad equity market indexes.

Figure 3 graphs coefficient estimates from rolling regressions for the SG Trend index. For each day, we estimate regression coefficients based on 5 years of trailing monthly returns. The red line shows point estimates of β^-, the slope coefficient during negative market environments. The green line shows point estimates of β^+, the slope coefficient during positive market environments. The blue line shows convexity, the difference between the estimates: $\kappa = \beta^+ - \beta^-$. The shaded areas around these lines indicate 95% confidence intervals. Ideally, we would like to see the point estimates for the betas and their confidence intervals to remain on either side of 0: negative for β^- and positive for β^+. This would indicate positive convexity for the returns. Strikingly, this was the case for many years early in the sample.

Starting about 10 years ago, however, the SG Trend returns apparently lost their positive convexity. Over the most recent decade, the convexity estimates have lost their statistical significance and have generally turned negative. In particular, market exposures during negative market environments now appear to be be positive instead of negative. This is the opposite of what many investors expect from CTA managers.

Figure 4 repeats this process for simulated portfolios based on short-, medium-, and long-term trend-following, respectively. As the figure shows, the long-term trend-following signals display very little evidence of positive convexity. In contrast, the short-term signals generally exhibit positive convexity. The medium-term signals appear to have lost positive convexity

Figure 3: Convexity in CTA Returns



The figure shows rolling estimates of market exposures and convexity for the SG Trend index. The SG Trend index is an index of the 10 largest CTA hedge funds.

The lines show estimates of market exposure during periods of positive excess returns for the market, β^+ in green, during periods of negative excess returns for the market, β^- in red, and convexity $\kappa = \beta^+ - \beta^-$ in blue. The shaded areas around the lines indicate plus or minus 2 standard errors around the estimates, covering roughly 95 percent confidence intervals.

The estimates are based on 5 years of rolling monthly returns. The return sample spans 264 months from January 2000 to December 2021.

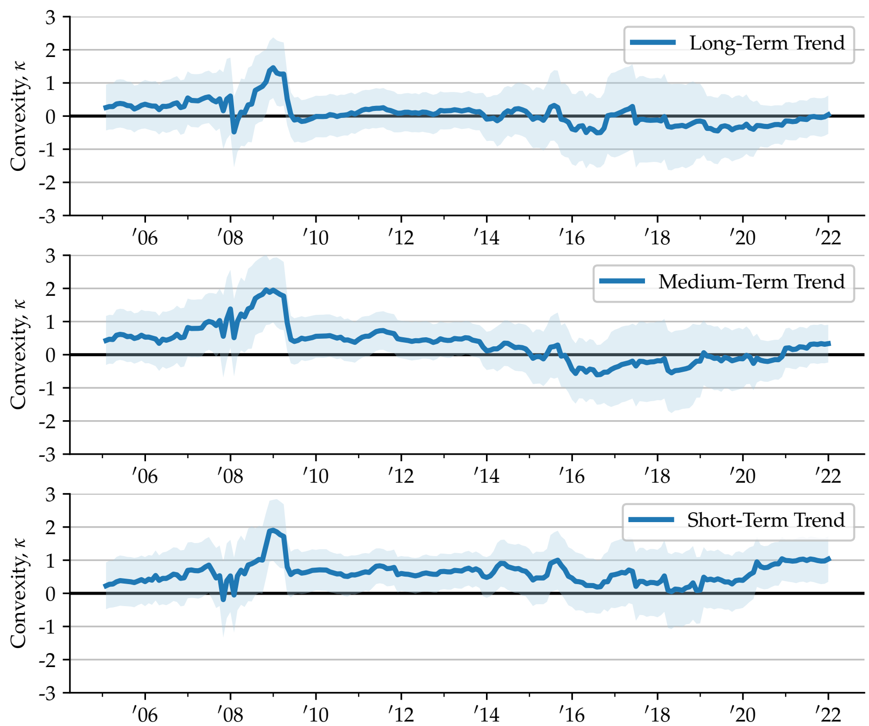
Source: Data received from Bloomberg and Société Générale. Internally prepared by Versor Investments.

Past performance is not indicative of future results. Performance results reflect the reinvestment of income. Commodity interest trading involves substantial risk of loss. These results are based on simulated or hypothetical returns that have inherent limitations. No representation is being made that any account is likely to achieve results similar to those shown. Please see additional important disclosures in the back.

over time. Since figure 1 shows that the managers in the SG Trend index have shifted from short-term to long-term trend-following signals, the difference in convexity among trend-following signals also explains why the SG Trend index has lost much of its earlier convexity.

This is striking evidence that positive convexity is not a feature of all trend-following investment styles. Investors interested in convexity must not presume that CTA funds universally provide returns with positive convexity. Clearly, differences in investment style matter.

Figure 4: Convexity in Trend-Following Returns



The figure shows rolling estimates of convexity for simulated trend-following strategies. The top panel shows results for long-term trend signals. The middle panel shows results for medium-term trend signals. The bottom panel shows results for short-term trend signals.

In each panel, the lines show estimates convexity $\kappa = \beta^+ - \beta^-$. The shaded areas around the lines indicate plus or minus 2 standard errors around the estimates, covering roughly 95 percent confidence intervals.

The estimates are based on 5 years of rolling monthly returns. The return sample spans 264 months from January 2000 to December 2021. The simulated returns are levered to the same annual volatility as the SG Trend index returns.

The short-term trend signals use lookback periods up to 3 months. The medium-term signals use lookback periods between 3 and 6 months. The long-term trend signals use lookback periods between 6 and 12 months.

The simulated returns include estimated transaction costs but no management fees.
Source: Data received from Bloomberg. Internally prepared by Versor Investments.

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5 Convex Signals

Trend-following signals are not the only signals that can generate positive convexity. Any signal that successfully predicts future returns allows us to be short during negative return environments and long during positive return environments. This generates returns with positive convexity in a long-short portfolio. Based on this logic, adding effective time-series signals to a trend-following portfolio can enhance the positive convexity of the portfolio.

Interestingly, cross-sectional, non-directional investment strategies can also generate positive convexity. An important source of this type of convexity is market volatility: Strategies that generate larger returns during volatile periods are likely to have positive convexity. Conversely, strategies that generate lower returns during volatile periods are likely to have negative convexity.

We now describe a collection of signals for futures trading and their convexity. For convenience, we follow the common approach of classifying signals into “value”, “momentum”, or “carry” groups. These classifications provide only a crude summary description since we generally include several signals in each category. Importantly, we explore both time-series and cross-sectional implementations of these signal groups.

5.1 Time-Series Signals

Time-series signals assess each asset in isolation and then establish a long or short position for that asset, depending on the signal. In this construction, the portfolio may be long many equity index futures or commodities at one point in time but short the same assets at another point in time. While a time-series portfolio may have low market exposures over the course of time, it can be materially net long or net short at any point in time.

Value

We use a range of asset-class-specific valuation criteria to assess each asset. If an asset appears expensive relative to its own past valuation measures, we establish a short position. Similarly, if an asset appears cheap relative to its own past valuation measures, we establish a long position. The valuation measures include inventories for commodities, real interest rates for fixed income, price-earnings ratios for equities, and purchasing power parity for currencies, among others. These trades generally profit when unusual valuations return to more normal levels.

Carry

In futures trading, carry signals use estimates of the (negative of the) slope near the front-end of the futures curve as return predictions. We calculate carry based on the difference between the the spot price and the near futures price, or the difference between the near and next futures prices. These differences are the equivalents of yield spreads. We go long assets with a positive yield spread and short assets with a negative yield spread. These trades generally profit when the yield spreads are large relative to return volatility.

Momentum

We use past returns as return predictions. The trend-following signals we described previously are the core of this signal family. We go long assets with positive past returns. These trades generally profit when past return trends continue into the future.

5.2 Cross-Sectional Signals

The time-series signals we describe above can also be converted into a cross-sectional implementation. For the cross-sectional version we compare signal values within asset classes to form market-neutral, long-short portfolios in each asset class. Creating separate cross-sections within each asset class makes the signal values more comparable. The differences between the time-series and cross-sectional constructions produce returns with relatively low correlations, even if the underlying signals are very similar.[5]

Value

We use the same asset-class-specific valuation criteria as above but now compare the values across contracts in the same asset class instead of over time. We establish large long positions in the most attractive asset, large short positions in the least attractive asset, and intermediate positions in the other assets, according to their valuation scores. As a result, a cross-sectional portfolio is market neutral at each point in time. These trades generally profit when the difference in valuation ratios compresses.

Carry

Similarly, we use the carry signals above to create long-short portfolios in each asset class. These trades generally profit when the differences in carry spreads compress.

Momentum

Finally, we use the momentum signals to create long-short portfolios in each asset class. These trades generally profit when the assets with the strongest past price trends continue to have the highest returns. Note that such a cross-sectional portfolio can earn positive returns even if all the assets in an asset class experience negative returns.

[5] For additional descriptions of the cross-sectional signals and portfolio construction, see Gurnani and Hentschel (2021).

Table 1: Convexity for Different Strategies

Portfolio	Full Sample		2002 – 2011		2012 – 2021	
Panel A: Trend-Following Signals						
SG Trend	0.36	(0.16)	0.52	(0.24)	0.04	(0.22)
LT Trend	0.07	(0.16)	0.13	(0.21)	-0.06	(0.27)
MT Trend	0.34	(0.16)	0.47	(0.22)	0.04	(0.25)
ST Trend	0.62	(0.16)	0.53	(0.21)	0.76	(0.25)
Panel B: Time-Series Signals						
Carry TS	0.17	(0.16)	0.52	(0.20)	-0.50	(0.26)
Value TS	0.30	(0.16)	0.21	(0.22)	0.53	(0.26)
Momentum TS	0.47	(0.16)	0.53	(0.22)	0.34	(0.24)
Panel C: Cross-Sectional Signals						
Carry CS	-0.04	(0.16)	0.25	(0.24)	-0.49	(0.18)
Value CS	0.32	(0.16)	0.57	(0.24)	-0.12	(0.22)
Momentum CS	-0.23	(0.17)	-0.33	(0.24)	0.10	(0.22)

The table shows convexity estimates for different investment strategies. The values in parentheses are standard errors for the estimates.

Panel A shows trend-following strategies. The SG Trend index is an index of the 10 largest CTA hedge funds. The LT Trend returns are based on trend-following signals with look-back periods from 6 to 12 months. The MT Trend returns are based on trend-following signals with look-back periods from 3 to 6 months. The ST Trend returns are based on trend-following signals with look-back periods up to 3 months.

Panel B shows time-series strategies based on value, carry, and momentum themes.

Panel C shows cross-sectional strategies based on value, carry, and momentum themes. The cross-sectional construction results in market-neutral portfolios in each asset class.

The full sample contains 240 monthly returns from January 2002 to December 2021. The pre-2011 sample contains 120 monthly returns from January 2002 to December 2011. The post-2011 sample contains 120 monthly returns from January 2012 to December 2021.

For comparison, the table levers all strategies to the same volatility as the SG Trend index.

The simulated returns include estimated transaction costs but no management fees.

Source: Data received from Bloomberg and Société Générale. Internally prepared by Versor Investments.

5.3 Convexity Over Time

We now show that many – but not all – of these signals have exhibited higher convexity than trend signals, especially in recent years. To conserve space, we summarize convexity in table 1 across 3 periods: the full sample, the period up to December 2011, and the period from January 2012 onward.

As we showed in figure 4 already, table 1 confirms that short-term trend-following signals consistently have the highest convexity among the trend-following signals. Among the other time-series signals, the Value strategies stand out with consistently high convexity. Similarly, time-series momentum has attractive convexity in all 3 sample periods. Among the cross-sectional signals, Value produces attractive convexity. Cross-sectional carry and momentum, however, are less consistent.

For simplicity, we show a small number of signal families that group a much larger number of underlying signals. If we look through to the underlying granular signals, however, we can find individual signals with attractive convexity in most of the groups. For example, it turns out that cross-sectional carry signals have poor convexity in currencies. However, cross-sectional carry signals have more attractive convexity in commodities. Nonetheless, the group averages shown in table 1 are broadly representative of the underlying signals.

6 The Sharpe-Convexity Frontier

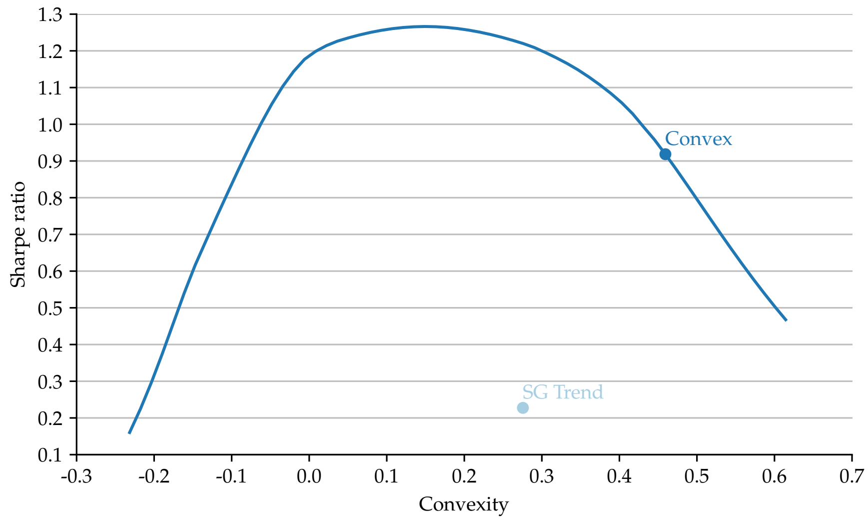
Given a collection of investment strategies, we can search for portfolios of these strategies that provide the strongest returns and highest convexity of returns. We formally search for such portfolios by finding the portfolio with the maximum Sharpe ratio for a given convexity. We can summarize the results of these searches on a "Sharpe ratio and convexity frontier" and then choose the portfolio with the most attractive combination of Sharpe ratio and convexity.

The Sharpe ratio is a natural measure of performance if the portfolio has a market beta close to zero. If the portfolio has material market exposures, we can hedge out this overall beta without affecting convexity.

Figure 5 shows an example Sharpe-convexity frontier. From the constituent strategies, we can build portfolios with different convexity levels. For a given level of convexity, we can then find the portfolio with the maximum Sharpe ratio. The blue line illustrates the resulting Sharpe-convexity frontier for the simulated strategies from table 1, excluding the SG Trend index.

The light blue circle on the frontier marks a convex portfolio for which we present further analysis. We intentionally select a portfolio with higher convexity and Sharpe ratio than the SG Trend index. However, there are several such portfolios along the frontier above and to the right of the SG Trend index. We choose a portfolio away from the end of the frontier.

Figure 5: Sharpe-Convexity Frontier



The figure shows the frontier of maximum Sharpe ratio for a given convexity based on combinations of 9 underlying strategies.

The simulated underlying strategies include: long-term, medium-term, and short-term trend following, time-series implementations of value, carry, and momentum, as well as cross-sectional implementations of value, carry, and momentum.

All of the simulated strategies are levered to the same long-term risk as the SG Trend index. The composite strategies on the frontier, however, have lower risk due to diversification.

The estimates of convexity and Sharpe ratios are based on 240 monthly returns from January 2002 to December 2021.

The figure marks the "convex" portfolio we use for further analysis. For comparison, the figure also marks the SG Trend index levered to the same risk as the convex portfolio. This leverage changes the convexity but not the Sharpe ratio.

The simulated returns include estimated transaction costs but no management fees.

Source: Data received from Bloomberg and Société Générale. Internally prepared by Versor Investments.

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Nearby portfolios are qualitatively similar. For comparison, the figure also shows the convexity and Sharpe ratio of the SG Trend index.[6]

6.1 Interpretation

The frontier allows us to separate the portfolio with the highest Sharpe ratio from all other portfolios with the same level of return convexity. Conversely, we can find the portfolio with the highest convexity for a given Sharpe ratio.

The mean-variance efficient frontier is a familiar analogue. There, we search for the portfolio with maximum return for a given risk level. Here,

[6]For comparison, we lever the SG Trend index to the same risk as the selected "convex" portfolio. This leverage affects the convexity but not the Sharpe ratio.

we search for the portfolio with the maximum Sharpe ratio for a given convexity. We use the Sharpe ratio as a performance measure, instead of returns, since the portfolio constituents can use leverage and may have different levels of risk and return. The Sharpe ratio removes the effects of this leverage and accounts for risk.

For the mean-variance frontier, we prefer portfolios with lower risk and higher returns. They are in the upper left of the familiar frontier diagram. For the Sharpe-convexity frontier, we prefer portfolios with higher convexity and higher Sharpe ratios. They are in the upper right of the frontier diagram.

For a portfolio employing a weighted average of underlying strategies, the portfolio convexity is the weighted average of the strategy convexities. This is directly analogous portfolio betas being equal to the weighted average beta of the constituents. In contrast, the Sharpe ratio of a portfolio is not a weighted average of the constituent Sharpe ratios, just like the portfolio risk is not a weighted average of the constituent risk levels.

As for the mean-variance frontier, the full Sharpe-convexity frontier contains dominated portfolios. For portfolios to the left of the maximum Sharpe ratio portfolio, we can find portfolios with the same Sharpe ratio and higher return convexity. We prefer these portfolios on the right side of the frontier. Figure 5 includes the left side for illustration.

Among the non-dominated portfolios on the right side of the frontier, however, there is not a single "best" portfolio. Here, there generally are tradeoffs between higher Sharpe ratios or higher convexity.

Conceptually, we can trace the Sharpe-convexity frontier by first finding the portfolio with maximum Sharpe ratio absent the convexity constraint. That portfolio is at the peak of the Sharpe-convexity constraint and has a convexity we can compute. From there, we can find additional portfolios with maximum Sharpe ratio subject to gradually increasing or decreasing required convexity levels. In that sense, the portfolios on either side of the peak are more constrained. As a result, they have lower Sharpe ratios.

From this mental calculation, we can see that the portfolios along the Sharpe-convexity frontier trace a path in mean-standard-deviation space. The path starts from the tangency portfolio without convexity constraints. As we require higher or lower convexity levels, the constrained frontier moves down in mean-standard-deviation space. As a result, the constrained Sharpe ratio falls.

Unlike the mean-variance frontier, the Sharpe-convexity frontier does not follow a particular functional form. The rate of decline from the peak Sharpe ratio can be different on the left and right side of the frontier. (We mostly care about the right side.) The steepness of the decline depends on the return characteristics of the available assets.

The frontier in figure 5 is an illustration built on the 9 signal portfolios described above. This is a fairly limited set of underlying strategies. The approach can handle an arbitrary number of strategies. With a larger number of strategies, the Sharpe-convexity frontier generally expands vertically and horizontally, making more attractive portfolios available. Of course, this also changes the trade-off between available Sharpe ratios and available convexity.

6.2 Optimization

To find the portfolio with maximum Sharpe ratio for a given level of convexity, we find the maximum Sharpe ratio portfolio subject to linear constraints on the portfolio weights. The constraints are:

- The portfolio has a specified level of convexity, say z.
- The weights sum to 1.
- The weights are positive.

Formally, we search for a portfolio $\boldsymbol{w}$ that solves

$$\begin{aligned} \max_{\boldsymbol{w}} \quad & \theta(\boldsymbol{w}) = \boldsymbol{w}'\boldsymbol{\mu}\left(\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}\right)^{-1/2} \\ \text{s.t.} \quad & \boldsymbol{w}'\boldsymbol{\kappa} = z \\ & \boldsymbol{w}'\boldsymbol{\iota} = 1 \\ & w_i \geq 0 \qquad \forall i, \end{aligned} \tag{7}$$

where $\boldsymbol{\mu}$ is the vector of expected excess returns associated with the strategies, $\boldsymbol{\Sigma}$ is the covariance matrix of strategy returns, $\boldsymbol{\kappa}$ is a vector of convexity levels associated with the strategies, z is a number we choose and hold fixed for a given optimization, $\boldsymbol{\iota}$ is a conformable vector of ones, and w_i is element i of the weight vector $\boldsymbol{w}$.

When we repeat this process for a range of convexity levels z, we find the portfolios along the Sharpe-convexity frontier. Generally, the most interesting range of convexity lies between the lowest convexity of the available strategies and the highest level of convexity of the available strategies.[7]

The final constraint rules out leverage. Like the familiar market beta, convexity is proportional to portfolio leverage. To avoid artificial increases in convexity due to leverage, we focus on optimizations that don't permit leverage.

[7] These are not strict limits. If we allow negative allocations or leverage, then the portfolios can attain convexity outside of the range of convexities associated with the individual strategies.

Like mean-variance optimizations with inequality constraints, these optimizations generally do not have analytical solutions. However, they can be solved iteratively using numerical methods. In particular, any portfolio optimization method that can find a maximum Sharpe ratio portfolio subject to standard portfolio constraints can solve the optimizations in equation 7.

Since the optimizations required for the Sharpe-convexity frontier are mean-variance optimizations, we can exploit experience with mean-variance optimizations in order to improve estimates of the Sharpe-convexity frontier. A well-known concern for mean-variance optimizations is that they may produce concentrated portfolios if they employ mean returns with large dispersion compared to the dispersion in risk characteristics. The corresponding portfolios have very large *ex ante* – but not *ex post* – Sharpe ratios. Naturally, the portfolios along the Sharpe-convexity frontier can become similarly concentrated under the same circumstances. Safeguards that are useful in mean-variance optimization are similarly effective here. Three approaches in wide-spread use are shrinkage of the expected returns, position limits, and shrinkage of the covariance matrix.[8] We use position limits by constraining the weights to be positive and less than 1.

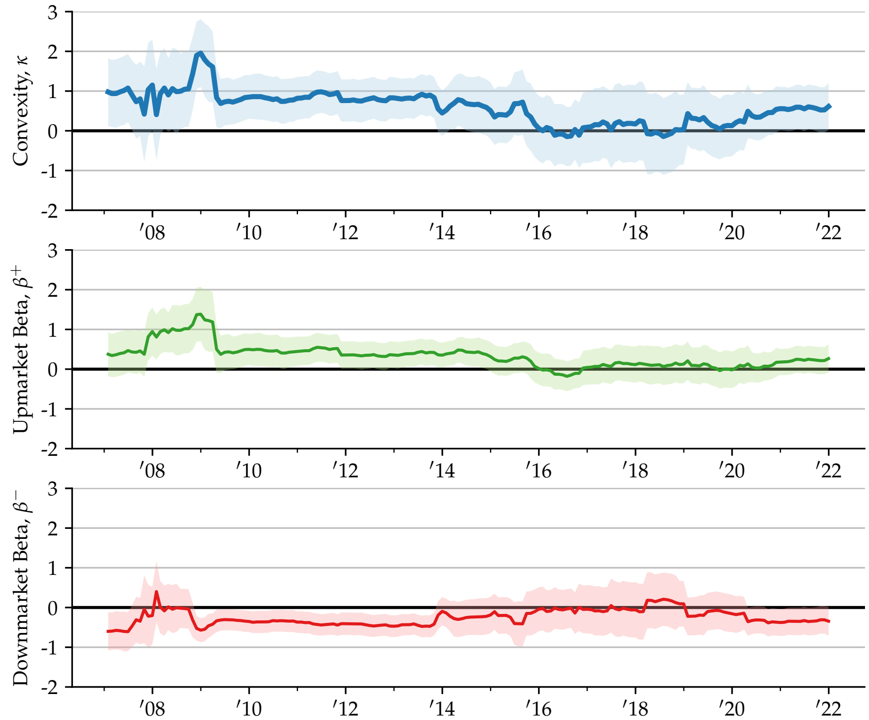
7 Convex Portfolios

Given the collection of signals from section 5, we can construct a composite signal that targets high returns with high convexity. As table 1 and figure 5 show, several of the new signals have higher convexity than trend-following signals. As a result, adding the new signals allows us to find portfolios with higher convexity than pure trend-following.

Figure 6 shows the rolling convexity estimates for the portfolio marked on the frontier in figure 5. Since we are interested in portfolios with high convexity, we intentionally did not choose the strategy with maximum Sharpe ratio. As the figure shows, however, we can start from the portfolio with maximum Sharpe ratio and materially increase convexity without large reductions in the Sharpe ratio. Comparing the estimates in figure 6 to those shown in figure 3 for the SG Trend index demonstrates that the enhanced portfolio consistently exhibits higher convexity. As the graphs show, the convex portfolio has materially more negative beta during negative equity markets than the SG Trend index.

[8]Common shrinkage methods for expected returns include statistical approaches based on James and Stein (1961), and financial approaches based on Black and Litterman (1992). Common shrinkage methods for the covariance matrix include Ledoit and Wolff (2004). Jagannathan and Ma (2003) show that position limits are closely related to covariance shrinkage.

Figure 6: Convex Portfolio



The figure shows rolling estimates of market exposures and convexity for a simulated convex portfolio. The lines show estimates of market exposure during periods of positive excess returns for the market, β^+ in green, during periods of negative excess returns for the market, β^- in red, and convexity $\kappa = \beta^+ - \beta^-$ in blue. The shaded areas around the lines indicate plus or minus 2 standard errors around the estimates, covering roughly 95 percent confidence intervals.

The portfolio blends a range of signals in order to find an attractive combination of Sharpe ratio and convexity. The signals are described in section 5. The portfolio construction along the Sharpe-convexity frontier is outlined in section 6.

The estimates are based on 5 years of rolling monthly returns. The return sample spans 240 months from January 2002 to December 2021.

The simulated returns include estimated transaction costs but no management fees.
Source: Data received from Bloomberg. Internally prepared by Versor Investments.

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Finally, table 2 summarizes the long-term performance characteristics of the convex trading strategy illustrated above and trend-following strategies. The table demonstrates that the convex portfolio has higher performance in addition to higher convexity. Of course, finding such a strategy was the purpose of constructing the Sharpe-convexity frontier. The table compares 5 strategies: the simulated convex portfolio, simulated long-term trend

Table 2: Return Characteristics for Convex Strategies

	Convex	LT Trend	MT Trend	ST Trend	SG Trend
Panel A: Full Sample					
Return	12.22	10.87	9.63	7.58	4.56
Risk	12.44	12.44	12.44	12.44	12.44
Sharpe	0.87	0.75	0.65	0.47	0.21
Convexity	0.62	0.07	0.34	0.61	0.37
Beta	-0.09	-0.12	-0.11	-0.16	-0.00
Max DD	-15.33	-16.77	-19.29	-24.08	-23.40
Panel B: 2002 – 2011					
Return	20.01	15.67	15.63	12.99	6.36
Risk	13.27	12.16	12.95	12.14	14.08
Sharpe	1.44	1.16	1.08	0.92	0.26
Convexity	0.72	0.13	0.46	0.53	0.55
Beta	-0.10	-0.12	-0.18	-0.14	-0.08
Max DD	-9.78	-16.24	-13.06	-11.22	-18.02
Panel C: 2012 – 2021					
Return	4.42	6.08	3.62	2.16	2.77
Risk	11.16	12.61	11.71	12.59	10.58
Sharpe	0.30	0.38	0.21	0.06	0.16
Convexity	0.44	-0.06	0.04	0.76	0.03
Beta	-0.02	-0.09	0.03	-0.16	0.12
Max DD	-15.33	-16.77	-19.29	-24.08	-23.40

The table shows performance characteristics for different simulated investment strategies and the SG Trend index. The columns show results for a convex strategy constructed from a collection of trend and non-trend signals, long-term trend following, medium-term trend following, short-term trend following, and the SG Trend index. The simulated strategies are levered to the same risk as the SG Trend returns.

All simulated strategies invest in about 100 liquid futures contracts across 4 major asset classes: equities, fixed income, commodities, and currencies.

The performance statistics in the rows are the annualized mean return in percentage points, the annualized standard deviation of returns in percentage points, the Sharpe ratio, the convexity of returns with respect to the S&P 500, the equity beta with respect to the S&P 500, and the maximum drawdown.

Panel A shows results for the full sample of 240 monthly returns from January 2002 to December 2021. Panel B shows results for 120 monthly returns from January 2002 to December 2011. Panel C shows results for 120 monthly returns from January 2012 to December 2021.

The simulated returns include estimated transaction costs but no management fees.

Source: Data received from Bloomberg and Société Générale. Internally prepared by Versor Investments.

following, simulated medium-term trend following, simulated short-term trend following, and the SG Trend index. As the table, shows, intelligently incorporating other signals into trend-following strategies can materially raise the Sharpe ratio and convexity compared to pure trend-following strategies.

The table shows a direct comparison based on identical simulation assumptions for all of the simulated strategies. To facilitate this comparison, all of the simulated strategies have been levered to the same long-term volatility as the SG Trend index.

In addition to the higher convexity, the convex portfolio has a higher Sharpe ratio than any of the purely trend-following strategies. This demonstrates that the Sharpe-convexity frontier can help us identify portfolios that increase convexity without sacrificing average returns.

8 Trend in 2021

Interestingly, trend-following strategies generally had a positive year in 2021, as we were writing this paper. For example, the SG Trend index returned 9.1 percent in 2021, its third-highest return in 10 years. While some have interpreted this as a revival of trend-following strategies, we caution that this performance likely was driven by material exposures to the least convex strategy components, which had exceptionally good performance in 2021.

The large exposures to long-term trend following shown in figure 3 suggest that some of the SG Trend returns can be attributed to positive market exposures during a period of rising equity markets. Unfortunately, the figure shows that these positive market exposures are not accompanied by positive convexity. Absent convexity, strategies with positive market exposures are likely to produce negative returns during periods of market drawdowns. Of course, this is not what most investors have historically expected from trend-following strategies.

To illustrate this point, we show the long-term convexity of several signals, their 2021 returns, and their long-term returns. Table 3 shows these summary statistics. Signals like long-term trends in equities have strongly negative convexity after the financial crisis but realized exceptional returns in 2021, as global equity markets collectively moved up.

Figure 7 illustrates a striking negative association between 2021 returns and signal convexity. The 2021 signal returns are marked in light blue. The light blue line shows the linear relation between 2021 returns and long-term signal convexity. In contrast, the dark blue markers show the long-term returns for the same signals. The dark blue line graphs the linear relation

Table 3: Convexity and 2021 Returns

	Convexity	Avg Return	2021 Return
Panel A: Developed Equities			
LT Trend	-0.40	1.15	21.31
MT Trend	-0.18	1.65	14.87
ST Trend	0.34	-1.29	2.45
Panel B: Commodities			
LT Trend	-0.18	1.91	2.44
MT Trend	0.10	1.74	18.04
ST Trend	0.24	-0.32	5.17
Panel C: Fixed Income			
LT Trend	0.15	6.52	-18.98
MT Trend	0.15	3.05	-15.03
ST Trend	0.37	4.17	7.09
Panel C: Developed Currencies			
LT Trend	0.11	-0.27	-7.91
MT Trend	-0.04	1.92	-2.88
ST Trend	0.10	0.52	-8.27

The table shows a range of signals, their long-term convexity, their long-term annualized average returns, and their 2021 returns. The full sample consists of monthly returns from January 2002 to December 2021.

All strategies simulate investments in about 100 liquid futures contracts across 4 major asset classes: equities, fixed income, commodities, and currencies. All strategies are levered to the same risk as the SG Trend index, 13.5 percent.

The simulated returns include estimated transaction costs but no management fees.

Source: Data received from Bloomberg. Internally prepared by Versor Investments.

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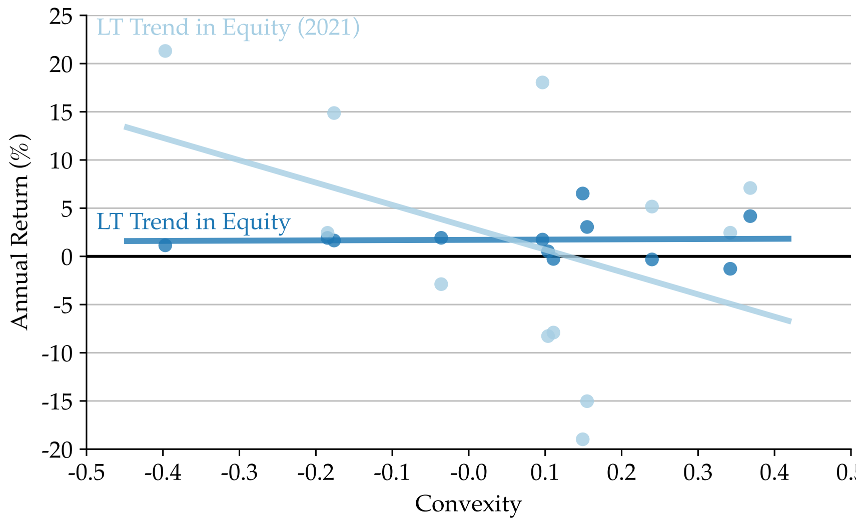
between long-term returns and long-term signal convexity. Clearly, 2021 demanded a material premium for positive convexity. Long-term, however, strategies with positive convexity do not earn lower average returns. The dark blue markers and line illustrate that there is no long-term association between average return and convexity.

While 2021 was a strong year for strategies with negative convexity, we caution that such strategies are unlikely to match investor expectations for trend-following portfolios. Most investors in trend-following portfolios expect these strategies to deliver positive convexity.

Relying on long-term trend signals in the most liquid asset classes, like equities and fixed income, is likely to lead to investor disappointment during periods of weak equity markets. Yet, those appear to be the most important

Figure 7: Returns and Convexity: Long-Term and 2021



The figure shows annualized average strategy returns for 2021 in light blue and from 2009 to 2021 in dark blue. All underlying return data are monthly. The horizontal axis shows the corresponding long-term convexities. Since convexity for several of these signal appears to be lower post 2008 than pre 2008, the figure uses data from January 2009 to December 2021.

All returns are levered to the same long-term volatility as the SG Trend index.

The lines indicate the best linear fit between average return and convexity over each of the two sample periods.

The text annotates the data points associated with long-term trend in equities, which has displayed material negative convexity since 2009.

The simulated returns include estimated transaction costs but no management fees.
Source: Data received from Bloomberg. Internally prepared by Versor Investments.

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signals for the the very large CTA funds that make up the SG Trend index. Futures strategies striving for positive convexity must allocate material risk to shorter-term trend signals and non-trend signals.

9 Summary

We show how to measure convexity in portfolio returns and demonstrate that the trend-following signals responsible for CTA hedge fund returns have lost some of their positive convexity over time. This is especially true for the long-term trend signals favored by the largest CTA hedge funds.

Since any successful timing signal generates returns with positive convexity, we show that portfolios that combine effective non-trend signals with some trend-following signals produce returns with superior returns and convexity compared to pure trend-following portfolios. We introduce the

Sharpe-convexity frontier that isolates the portfolios with the maximum Sharpe ratio for a given level of convexity. This frontier allows investors to make efficient choices among portfolios with high Sharpe ratios and high convexity. Although these portfolio choices generally require a tradeoff between Sharpe ratios and convexity, the frontier separates the most attractive portfolios from the rest.

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The **SG Trend Index** represents an index of the 10 largest trend-following CTA hedge funds. The funds must be open to new investments and report daily returns.